

# On the behavior of the nuclear spectral function at high momentum and removal energy

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Received: 1 December 1998 / Revised version: 19 January 1999

Communicated by W. Weise

**Abstract.** We propose a procedure to extrapolate the nuclear spectral function  $P(|\mathbf{k}|, E)$  obtained from nonrelativistic many-body theory to large values of three-momentum and removal energy. Our approach is based on phenomenological information extracted from both soft hadron-nucleus interactions, in the regime where the proton inclusive spectrum is dictated by Regge asymptotic, and deep-inelastic lepton-nucleus collisions. The extrapolated  $P(|\mathbf{k}|, E)$  is used to compute the semi-inclusive spectra of backward protons produced in electron-nucleus scattering.

**PACS.** 13.60.Le Meson production – 25.30.Fj Inelastic electron scattering to continuum – 25.30.Rw Electroproduction reactions

The knowledge of the spectral function  $P(|\mathbf{k}|, E)$ , giving the probability to find a nucleon of momentum  $|\mathbf{k}|$  and removal energy  $E$  inside a nucleus, is a prerequisite for the theoretical description of a number of reactions involving nuclear targets. For both infinite nuclear matter and light nuclei, with mass number  $A \leq 4$ , it is possible to carry out accurate calculations of  $P(|\mathbf{k}|, E)$  starting from a realistic nuclear hamiltonian, fitted to nucleon-nucleon scattering data and to the properties of few-nucleon bound states [1, 2]. In the case of medium-heavy nuclei, quantitative estimates of the spectral function and its energy integral, the momentum distribution  $n(|\mathbf{k}|)$ , can also be obtained, using the local density approximation [3].

The spectral functions resulting from realistic many-body calculations contain information on both the nuclear mean field (at low  $|\mathbf{k}|$  and  $E$ ), and short-range nucleon-nucleon correlations (at high  $|\mathbf{k}|$  and  $E$ ). Typically, only about 70% of the nucleons are in the states of low  $|\mathbf{k}|$  and low  $E$ , that can be described by mean field single-particle wave functions, while the remaining 30% are in a correlated state with another nucleon, mainly on account of the one-pion-exchange tensor force and the short-range repulsion of the nucleon-nucleon interaction. The mean field ( $P_0(|\mathbf{k}|, E)$ ) and correlation ( $P_B(|\mathbf{k}|, E)$ ) contributions to

the spectral function can be singled out rewriting the full  $P(|\mathbf{k}|, E)$  in the form

$$P(|\mathbf{k}|, E) = P_0(|\mathbf{k}|, E) + P_B(|\mathbf{k}|, E) . \quad (1)$$

Strongly correlated nucleons play a very important role in many processes. For example, the production of fast backward hadrons in semi-inclusive lepton-nucleus reactions, in the kinematical region forbidden to scattering off a free nucleon, is mostly due to nucleon-nucleon correlations [4, 5]. While many-body calculations typically provide a description of the correlation tail of  $P(|\mathbf{k}|, E)$  up to  $k_{max} \sim .7$  GeV/c and  $E_{max} \sim .6$  GeV, the theoretical analysis of the available spectra of lepton-produced backward hadrons carrying large momentum [6] requires the knowledge of the nuclear spectral function at larger values of  $|\mathbf{k}|$  and  $E$  [4]. In this note, we propose a simple phenomenological procedure that allows one to extrapolate the correlation contribution,  $P_B(|\mathbf{k}|, E)$ , beyond the region covered by nuclear many-body theory.

Let us consider a process in which a four-momentum  $q \equiv (\nu, \mathbf{q})$  is transferred to a nuclear target. Our starting point is the relationship between the nuclear spectral function and the function  $f_A(z)$ , yielding the distribution of the nucleons in the target as a function of the relativistic invariant variable  $z$ , defined as

$$z = \frac{M_A}{m} \frac{(kq)}{(PAq)} . \quad (2)$$

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In the above equation  $m$  is the nucleon mass, whereas  $P_A \equiv (M_A, 0)$ ,  $M_A$  being the target mass, and  $k \equiv (k_0, \mathbf{k})$  denote the initial nuclear and nucleon four-momentum in the target rest frame, respectively. The distribution function  $f_A(z)$  can be written in a general fashion as

$$f_A(z) = z \int d^4k S(k) \delta\left(z - \frac{M_A}{m} \frac{(kq)}{(PAq)}\right), \quad (3)$$

where  $S(k)$  is the relativistic function describing the nuclear vertex with an outgoing nucleon of four-momentum  $k$  (see, e.g., [7]).  $S(k)$  can be approximated by the nonrelativistic spectral function according to [8]

$$S(k) = \left(\frac{m}{k_0}\right) P(|\mathbf{k}|, E), \quad (4)$$

with

$$k_0 = M_A - \left[(M_A - m + E)^2 + |\mathbf{k}|^2\right]^{1/2}. \quad (5)$$

Note that the above definition implies that  $f_A(z)$  also depends upon  $Q^2 = -q^2$ , as pointed out in [8]. However, in the following we will be assuming that the Bjorken limit ( $Q^2, \nu \rightarrow \infty$ ) is applicable, so that  $z$  can be related to the light-cone component of the nucleon four-momentum through  $z = (k^+/m) = (k_0 - k_z)/m$  ( $k_z = (\mathbf{k}\mathbf{q})/|\mathbf{k}||\mathbf{q}|$ ) and the  $Q^2$ -dependence of  $f_A(z)$  disappears.

Substituting (4) into (3) we can rewrite  $f_A(z)$  in terms of  $P(|\mathbf{k}|, E)$  as

$$f_A(z) = 2\pi m z \int_{E_{min}}^{\sqrt{s}-M_A} dE \times \int_{k_{min}(z,E)}^{\infty} d|\mathbf{k}| |\mathbf{k}| \left(\frac{m}{k_0}\right) P(|\mathbf{k}|, E) \quad (6)$$

where  $E_{min}$  is the minimum energy required to remove a nucleon from the target nucleus,  $s = (P_A + q)^2$  and (for simplicity, we work in the infinite nuclear matter limit,  $M_A \rightarrow \infty$ , in which the kinetic energy of the recoiling nucleus becomes vanishingly small)

$$k_{min}(z, E) = |m(1-z) - E|. \quad (7)$$

The above equations imply that the large  $z$  behavior of  $f_A(z)$  is dictated by the high  $|\mathbf{k}|$  tail of  $P(|\mathbf{k}|, E)$ , i.e. from its correlation part  $P_B(|\mathbf{k}|, E)$ . For example, at  $z > 1.7$  only momenta larger than  $k_{max} \sim .7$  GeV/c, contribute to the integral of (6).

In [4] we have developed a procedure to evaluate the asymptotic behavior of  $f_A(z)$  without making use of the nuclear spectral function. Within this approach, based on ideas originally proposed in [9,10],  $f_A(z > 1)$  can be written as a sum of distributions  $f_n(z/n)$ , describing clusters of  $n$  ( $n \geq 2$ ) strongly correlated nucleons, whose asymptotic behavior as  $z \rightarrow n$  can be evaluated for any value of  $Q^2$ . We will show that using the large  $z$  behavior of  $f_A(z)$  resulting from the approach of [4] and (6) one can extract information on the behavior of  $P(|\mathbf{k}|, E)$  at very large  $|\mathbf{k}|$  and  $E$ .

For any given  $|\mathbf{k}|$ , the nuclear matter  $P_B(|\mathbf{k}|, E)$  of [1] exhibits a bump located at  $E \sim E_k = \sqrt{|\mathbf{k}|^2 + m^2} - m$ , the energy needed to remove a nucleon of momentum  $\mathbf{k}$  belonging to a strongly correlated pair of vanishing total momentum, whose width increases with  $|\mathbf{k}|$ . A remarkably accurate fit to the behavior of the correlation contribution to the calculated spectral function has been obtained in [11], where  $P_B(|\mathbf{k}|, E)$  has been written in the form

$$P_B(|\mathbf{k}|, E) = n_B(|\mathbf{k}|) F(|\mathbf{k}|, E), \quad (8)$$

with

$$n_B(|\mathbf{k}|) = \int_{E_{thr}}^{\infty} dE P_B(|\mathbf{k}|, E), \quad (9)$$

$E_{thr}$  being the minimum energy required to remove a nucleon pair. The function  $F(|\mathbf{k}|, E)$  is defined as

$$F(|\mathbf{k}|, E) = N_k \exp \left\{ \frac{\left[ \sqrt{m(E - E_{thr})} - \sqrt{m(E_k - E_{thr})} \right]^2}{2\sigma_k^2} \right\}, \quad (10)$$

where  $\sigma_k$  is related to the width  $\Gamma_k$  through

$$\Gamma_k = 4 \sigma_k \sqrt{(2 \ln 2)(E_k/m)} = \langle E_B \rangle + E_k, \quad (11)$$

$\langle E_B \rangle$  being the average removal energy associated with  $P_B(|\mathbf{k}|, E)$  (using the spectral function of [1] one finds  $\langle E_B \rangle \sim 40$  MeV). The normalization constant  $N_k$  is chosen in such a way as to fulfill the sum rule

$$\int_{E_{thr}}^{\infty} dE F(|\mathbf{k}|, E) = 1, \quad (12)$$

which in turn guarantees the overall normalization of  $P_B(|\mathbf{k}|, E)$ .

Our extrapolation procedure is based on the assumption that at very large values of  $|\mathbf{k}|$  and  $E$  the energy dependence of  $P_B(|\mathbf{k}|, E)$  is still dominated by the contribution associated with the removal of a correlated nucleon pair, and can be described by  $F(|\mathbf{k}|, E)$  of (10). As a consequence, the extrapolation of  $P(|\mathbf{k}|, E)$  reduces to the extrapolation of  $n_B(|\mathbf{k}|)$ .

Substitution of (8) into (6) leads (after inversion of the integration order) to the following expression for the correlation contribution to  $f_A(z)$  at large  $z$ :

$$f_A^B(z) = 2\pi m z \int_{k_{min}(z, E_{thr})}^{\infty} d|\mathbf{k}| \Phi(|\mathbf{k}|), \quad (13)$$

with

$$\Phi(|\mathbf{k}|) = |\mathbf{k}| n_B(|\mathbf{k}|) \int_{E_{thr}}^{|\mathbf{k}| - m(z-1)} dE \left(\frac{m}{k_0}\right) F(|\mathbf{k}|, E), \quad (14)$$

implying in turn

$$\phi'(z) = \frac{d}{dz} \left( \frac{f_A^B(z)}{z} \right) = -2\pi m^2 \Psi(z), \quad (15)$$

where

$$\Psi(z) = m \int_{E_{thr}}^{\infty} dE \frac{E + m(z-1)}{m-E} n_B(E + m(z-1)) \times F(E + m(z-1), E). \quad (16)$$

Knowing the distribution function  $f_A^B(z)$ , which can be obtained from the approach developed in [4] for any values of  $z > 1$ , the function  $\Psi(z)$  appearing in left hand side of the above equation can be readily evaluated. Hence, inversion of (16) at  $z > 1.7$  would immediately yield  $n_B(|\mathbf{k}|)$  at  $|\mathbf{k}| > k_{max}$ . In general, however, (16) cannot be inverted analytically. To circumvent this problem we have used a parametrized  $n_B(|\mathbf{k}|)$  in the right hand side of (16), with the values of the parameters adjusted in such a way as to reproduce  $\Psi(z)$  evaluated according to [4].

At  $k_F < |\mathbf{k}| < k_{max}$ ,  $k_F \sim .25$  GeV/c being the Fermi momentum, the behavior of the momentum distribution of [1] is nearly exponential, and can be accurately approximated using

$$n_0(|\mathbf{k}|) = G_1 \exp[-(B_1 |\mathbf{k}|)^\alpha], \quad (17)$$

with  $G_1 = 3.30$  (GeV/c)<sup>-3</sup>,  $B_1 = 6.2$  (GeV/c)<sup>-1</sup> and  $\alpha = 1.14$ .

The most natural choice is to use a similar form to calculate the right hand side of (16) for  $z > 1.7$ . It should be noted, however, that the integrand in (16) has a singularity at  $E = m$ . This problem has been taken care of inserting a cutoff to guarantee that  $n_B(|\mathbf{k}|)$  vanish at  $|\mathbf{k}| = mz$ . The parametrization of  $n_B(|\mathbf{k}|)$  providing the best fit to  $\Psi(z)$  turns out to be

$$n_B(|\mathbf{k}|) = \exp \left[ -\beta \left( \frac{|\mathbf{k}| - k_0}{\Lambda - |\mathbf{k}|} \right) \right] n_0(|\mathbf{k}|), \quad (18)$$

where  $\beta = .001$ ,  $k_0 \sim k_{max}$ ,  $\Lambda = mz_0 = m + k_0 + E_{thr}$  and

$$n_0(|\mathbf{k}|) = G_1 \exp[-(B_1 |\mathbf{k}|)^\alpha] + G_2 \exp(-B_2 |\mathbf{k}|), \quad (19)$$

with  $G_1 = 2.90$  (GeV/c)<sup>-3</sup>,  $B_1 = 6.08$  (GeV/c)<sup>-1</sup>,  $G_2 = -21.2$  (GeV/c)<sup>-3</sup> and  $B_2 = 26.7$  (GeV/c)<sup>-1</sup>. With the above choice of  $n_B(|\mathbf{k}|)$ , the right hand side of (16) is well defined and can be calculated for any  $z_0 < z < 2$ , the upper limit being dictated by the fact that (10) is only appropriate to describe correlated two-nucleon clusters.

The results of numerical calculations show that substituting  $n_B(|\mathbf{k}|)$  defined by (18)-(19) into (14), and using (13), the distribution function  $f_A^B(z)$  of [4] is reproduced to a very high accuracy in the range  $1.7 < z < 2$ .

A more straightforward procedure to extract  $n_B(|\mathbf{k}|)$  from (15) can be obtained making the rather drastic assumption that in (16) the function  $F(|\mathbf{k}|, E)$  can be replaced by a  $\delta$ -function:

$$F(E + m(z-1), E) = \delta(E - (\sqrt{(E + m(z-1))^2 + m^2} - m)). \quad (20)$$

Using (20) the  $E$  integration in (16) can be readily carried out and substitution of the result into (15) leads to:

$$n_B(k_s) = \frac{1}{2\pi m^2 k_s} \frac{(3-z^2)(2-z)}{(2-z)^2 + 1} \phi'(z), \quad (21)$$

with  $k_s = m(z-1)(z-3)/(2(z-2))$ . Eq.(21) gives  $n_B(|\mathbf{k}|)$  in terms of  $\phi'(z)$  for any values of  $z$  in the range  $1 < z < \sqrt{3}$ .

Let us now focus on the calculation of the left hand side of (15). According to [4], at large values of its argument  $f_A^B(z)$  can be written as a sum, whose terms describe the contributions associated with strongly correlated  $n$ -nucleon clusters:

$$f_A^B(z) = \sum_{n=2}^A f_n \left( \frac{z}{n} \right), \quad (22)$$

the  $n$ -th term in the sum being defined for  $1 < z < n$ . Within this approach the calculation of  $f_A^B(z)$  reduces to the calculation of the relevant  $f_n(z/n)$ 's, corresponding to the lowest values of  $n$  (typically  $n = 2$  and  $3$ ). The analysis of nuclear fragmentation in hadron-nucleus collisions carried out in [9,10] shows that, at low  $Q^2$ , the distribution of colorless three-quark systems in a  $3n$ -quark cluster,  $T_n(z/n)$ , exhibits true Regge asymptotic behavior as  $z \rightarrow n$ . The results of [9,10] provide a satisfactory description of the inclusive spectra of high-momentum protons and mesons emitted backward in proton-nucleus collisions. However, the sizeable  $Q^2$ -dependence exhibited by  $f_A(z)$  at low  $Q^2$  [8] suggests that the nonperturbative  $Q^2$ -dependence of  $T_n(z/n)$  has to be carefully taken into account. Starting from the small  $Q^2$  behavior, which can be described within the framework of Regge theory, the asymptotic  $T_n(z/n)$  at  $z \rightarrow n$  and large  $Q^2$  can be obtained from the distribution of valence quarks inside a cluster of  $n$  strongly correlated nucleons, which can in turn be written in terms of the relativistic invariant phase-space volume available to a quark in a nucleon [4].

The function  $T_n(z/n)$  can be interpreted as the distributions of *effective nucleons* within a strongly correlated cluster. Therefore, assuming that the valence quark distribution inside these *effective nucleons* is the same as in ordinary nucleons, the quantity  $\tilde{T}_n(z/n) = w_n T_n(z/n)$ ,  $w_n$  being the probability of finding an  $n$ -body cluster [4], can be identified with  $f_n(z/n)$  of (22). The validity of this approximation, which allows one to effectively take into account nuclear excitations, is supported by the results of calculations of the valence quark distributions inside nucleons and baryonic resonances [12,13].

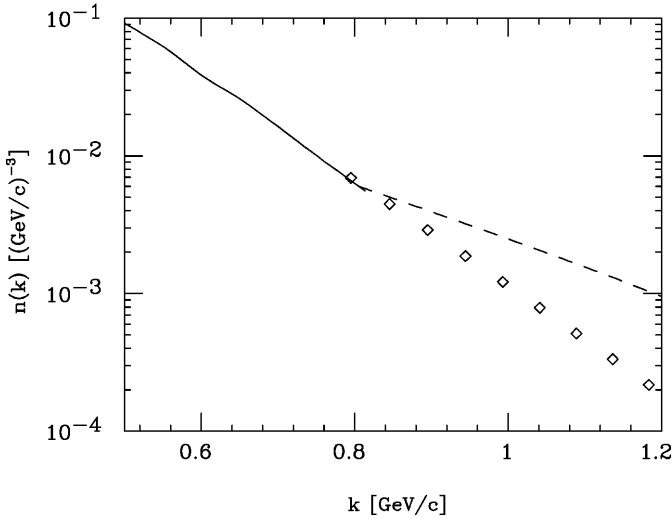
Writing the distribution of valence quarks inside a nucleon at large  $Q^2$  in the form

$$f_{q_v}^N(z) = C_N z^{a_N} (1-z)^{b_N}, \quad (23)$$

where  $C_N$  is a normalization constant,  $a_N = -\alpha_R(0) = 1/2$ ,  $\alpha_R(0)$  being the intercept of the Regge trajectory, and  $b_N \sim 2.8 - 3.2$ , the behavior of  $\tilde{T}_n(z/n)$  as  $z \rightarrow n$  can be obtained in closed form [14]. Under the assumption  $\tilde{T}_n(z/n) = f_n(z/n)$  we can use this result and write:

$$f_n \left( \frac{z}{n} \right) = D_n \left( \frac{z}{n} \right)^{A_n} \left[ 1 - \left( \frac{z}{n} \right) \right]^{g_n}, \quad (24)$$

with  $g_n = (a_N + b_N + 2)(n-1) - 1$  and  $A_n = a_N + b_N + 1$ , whereas the coefficients  $D_n$  can be obtained from the



**Fig. 1.** The nucleon momentum distribution in infinite nuclear matter. The solid line corresponds to the  $n_B(|\mathbf{k}|)$  of [1], whereas the diamonds show the large  $|\mathbf{k}|$  extrapolation parametrized according to (18)-(19). The dashed line has been obtained using the  $\delta$ -function approximation of (20)

quark distribution in the  $n$ -nucleon cluster, evaluated as in [4]. The details of the derivation of (24) are given in the Appendix.

Substituting the  $f_n(z/n)$ 's with  $n = 2, 3$  and 4, calculated from (24), into (22), one can easily obtain both  $f_A^B(z)$  and its first derivative at  $z < 4$ . The resulting  $\phi'$ , defined as in (15), can be written:

$$\phi'(z) = I_2 + I_3 + I_4, \quad (25)$$

where  $I_n(z)$  is defined for  $z < n$  and

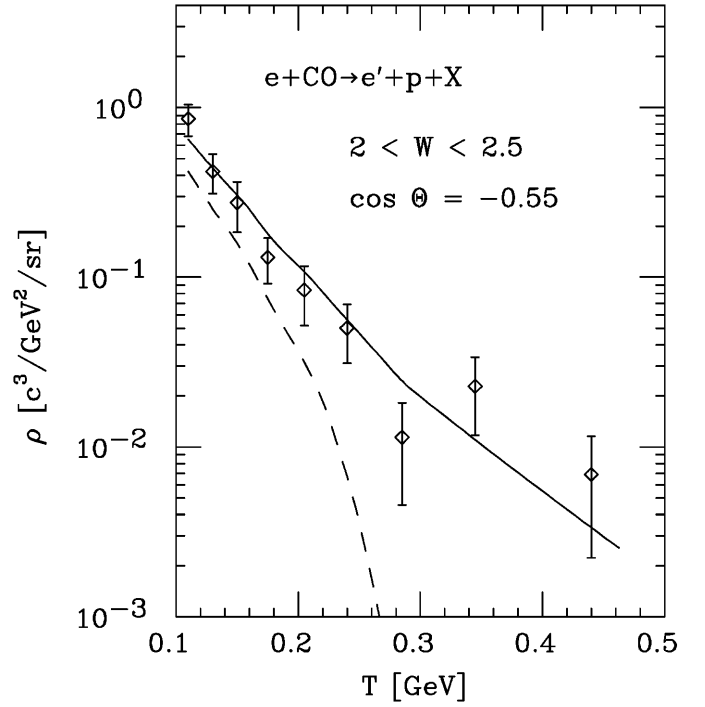
$$I_2 = \frac{1}{2} D_2 \left(\frac{z}{2}\right)^{A_2-2} \left[1 - \left(\frac{z}{2}\right)\right]^{g_2-1} \times \left\{ (A_2 - 1) \left[1 - \left(\frac{z}{2}\right)\right] - g_2 \left(\frac{z}{2}\right) \right\}, \quad (26)$$

$$I_3 = \frac{1}{3} D_3 \left(\frac{z}{3}\right)^{A_3-2} \left[1 - \left(\frac{z}{3}\right)\right]^{A_3-2} \left[1 - \left(\frac{z}{3}\right)\right]^{g_3-1} \times \left\{ (A_3 - 1) \left[1 - \left(\frac{z}{3}\right)\right] - g_3 \left(\frac{z}{3}\right) \right\} \quad (27)$$

$$I_4 = \frac{1}{4} D_4 \left(\frac{z}{4}\right)^{A_4-2} \left[1 - \left(\frac{z}{4}\right)\right]^{g_4-1} \times \left\{ (A_4 - 1) \left[1 - \left(\frac{z}{4}\right)\right] - g_4 \left(\frac{z}{4}\right) \right\}. \quad (28)$$

With  $\phi'(z)$  given by (25)-(28), the function  $\Psi(z)$  appearing in the left hand side of (16) can be readily evaluated. Hence, the momentum distribution can be obtained using either (16) and the fitting procedure or the  $\delta$ -function ansatz leading to (21).

Figure 1 shows the nuclear matter momentum distribution  $n(|\mathbf{k}|)$  of [1], calculated for  $|\mathbf{k}| \leq 0.8$  GeV/c (solid line), together with the values obtained from the fit to



**Fig. 2.** Kinetic energy spectra of protons emitted at angle  $\Theta$  in the semi-inclusive reaction  $e + CO \rightarrow e' + p + X$ . The dashed curve has been obtained using the nucleon momentum distribution of [1] at  $|\mathbf{k}| < k_{max} \sim 0.8$  GeV/c and assuming  $n_B(|\mathbf{k}| > k_{max}) = 0$ , whereas the solid line shows the results obtained using the large  $|\mathbf{k}|$  extrapolation, parametrized according to (18)-(19), at  $|\mathbf{k}| > k_{max}$ . The experimental data are taken from [6]

$\phi'(z)$  at  $|\mathbf{k}| > 0.8$  GeV/c (diamonds). It appears that the extrapolated tail of  $n(|\mathbf{k}|)$  is close to a simple exponential dependence. The results obtained using the  $\delta$ -function approximation, which provides an upper bound to  $n_B(|\mathbf{k}|)$  at  $|\mathbf{k}| > 0.8$  GeV/c are also shown in Fig. 1 (dashed line). It is apparent that the energy spread of the strength is very important and cannot be disregarded.

To test the extrapolated momentum distribution, we have calculated the kinetic energy spectrum of protons emitted at backward angle in the semi-inclusive  $e + A \rightarrow e' + p + X$  reaction, in the kinematics of the SLAC data of [6], using [4]

$$\rho_{eA \rightarrow e'pX}(x, Q^2, z) = \frac{I_{eN}}{I_{eA}} n_B(|\mathbf{k}|(z)) F_2^N(x, Q^2). \quad (29)$$

In the above equation,  $F_2^N(x, Q^2)$ ,  $x$  being the Bjorken scaling variable, is the nucleon structure function, while  $I_{eN}$  and  $I_{eA}$  denote the fluxes associated with scattering off an isolated nucleon and the nuclear target, respectively. In Fig. 2 the spectrum obtained using the nucleon momentum distribution of [1] at  $k < k_{max} \sim 0.8$  GeV/c and setting  $n_B(|\mathbf{k}| > k_{max}) = 0$  is compared to that obtained using  $n(|\mathbf{k}| > k_{max})$  obtained from the fit to  $f_A^B(z)$  at  $z \geq 1.7$ . It appears that the extrapolated  $n_B(|\mathbf{k}|)$ , needed to describe the spectrum at large proton energy ( $T \geq 0.3$

GeV), also provides a much better description of the data at lower  $T$ .

In conclusion, we have proposed a simple phenomenological procedure that allows one to extrapolate the nuclear matter spectral function beyond the region described by nonrelativistic many-body calculations. The main assumption needed to extract  $n_B(|\mathbf{k}|)$  from the relationship between the spectral function and the distribution function  $f_A(z)$ , i.e. the assumption that  $P_B(|\mathbf{k}|, E)$ , exhibits the  $E$ -dependence associated with the removal of a nucleon belonging to a strongly correlated pair, appears to be adequate in the range of momentum and removal energy relevant to our analysis, corresponding to  $z < 2$ . The importance of the description of the  $E$ -dependence is clearly shown by the fact that the oversimplified  $\delta$ -function ansatz leads to a severely overestimated momentum distribution. The numerical results shown in Figs. 1 and 2 suggest that our approach can be used to quantitatively investigate reactions sensitive to the very high momentum components of the nuclear wave function.

This work has been encouraged and supported by the Russian Foundation of Fundamental Research. We gratefully acknowledge many helpful discussions with A. Fabrocini.

## Appendix A

In [4] it has been shown that, assuming that the distribution of valence quarks inside an isolated nucleon can be described by

$$f_{q_v}^N(z) \sim z^{a_N}(1-z)^{b_N}, \quad (\text{A1})$$

the corresponding distribution inside a strongly correlated  $n$ -nucleon cluster, obtained from the overlap of the phase-space volumes available to a quark inside a nucleon, takes the form

$$f_{q_v}^{(n)}\left(\frac{z}{n}\right) = C_n \left(\frac{z}{n}\right)^{a_N} \left[1 - \left(\frac{z}{n}\right)\right]^{[b_N + (a_N + b_N + 2)(n-1)]}, \quad (\text{A2})$$

where the coefficient  $C_n$  can be written in terms of beta-functions.

The function  $f_{q_v}^{(n)}(z/n)$  can also be given in terms of the distribution of colorless three-quark systems within a  $3n$ -quark cluster,  $\tilde{T}_n(z')$ , using Mellin's convolution formula:

$$f_{q_v}^{(n)}\left(\frac{z}{n}\right) = \int_{(z/n)}^1 \tilde{T}_n(z') f_{q_v}^{3Q}\left(\frac{1}{n} \frac{z}{z'}\right) \frac{dz'}{z'}, \quad (\text{A3})$$

where  $f_{q_v}^{3Q}(z/nz')$  denotes the distribution of valence quarks inside the colorless three-quark system.

Assuming that at large  $z/(nz')$   $f_{q_v}^{3Q}(z/nz')$  is approximately the same as the distribution inside a nucleon, e.g. assuming  $f_{q_v}^{3Q} \sim f_{q_v}^N$ , and using (A1), (A3) can be inverted to obtain  $\tilde{T}_n(z)$  in the form:

$$\tilde{T}_n(z) = D_n z^{A_n} (1-z)^{g_n}, \quad (\text{A4})$$

with  $D_n$ ,  $A_n$  and  $g_n$  given in terms of the constants  $C_n$ ,  $a_N$  and  $b_N$  appearing in (A2).

Within this approximation (A4) reduces to (24), since  $\tilde{T}_n$  can be identified with  $f_n$ , which can in turn be interpreted as the distribution of colorless three-quark systems inside of a strongly correlated  $n$ -nucleon cluster.

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